Announcements

1) New HW up later today, due Wednesday next week.

2) Sections to be covered 2.2, 2.3, 3.1, 3.2, 3.3

Properties of Addition

Let A, B be $m \times n$ matrices.

1) Commutativity

$$A+B = B+A$$

2) Associativity

$$(A+B)+C = A+(B+C)$$

3) Identity

Let $O_{m,n}$ be the $m \times n$ matrix with all entries equal to zero.

Then

$$A + O_{m,n} = A$$

We call $O_{m,n}$ the additive identity for $m \times n$ matrices.

4) Additive Inverse

Let $-A$ be the matrix whose entries are the negatives of the entries of A. Then

$$A + (-A) = O_{m,n}$$

When adding $-B$ to A,

we usually write

$A-B$ instead of $A+(-B)$.

The matrix $-A$ is called the

additive inverse of A.

You are free to use all

these facts without justification.

Properties of Multiplication

Let A be an $m \times n$ matrix, B be an $n \times k$ matrix, and C be a $k \times j$ matrix.

1) Associativity

$$\underbrace{(A \cdot B)}_{m \times k} \cdot \underbrace{C}_{k \times j} = \underbrace{A}_{m \times n} \cdot \underbrace{(B \cdot C)}_{n \times j}$$

2) Distributivity

Let D be a $n \times k$ matrix.

Then

$$A(B+D) = A \cdot B + A \cdot D$$

$$(B+D)C = B \cdot C + D \cdot C$$

3) Identity $(n \times n)$

Let I_n denote the $n \times n$ matrix whose entries are 1 on the diagonal, zeros everywhere else.

For example,

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

etc.

Then if $m=n$,

$$A \cdot I_n = I_n \cdot A = A$$

Warning: Suppose A and B are $n \times n$ matrices. Is **NOT** always true that

$$AB = BA.$$

Moreover, multiplicative inverses do not necessarily exist; that is, given A, there is **NOT** always a B with

$$AB = BA = I_n$$

Example 1:

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$B = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$.

$$AB = \begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

These are not equal, so $AB \neq BA$.

Example 2!

Let $A = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$.

Then if $AB = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

$B = \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix}$, we have

$2b_{1,1} + 3b_{2,1} = 1$ (1,1) entry

$2b_{1,2} + 3b_{2,2} = 0$ (1,2) entry

$0 = 0$ (2,1) entry

$0 = 1$

We see that A cannot have a multiplicative inverse.

Q: How do you tell when an $n \times n$ matrix has a multiplicative inverse?

A: Determinants!

The only time an $n \times n$ matrix has a multiplicative inverse is when its determinant is nonzero!